CharisMATHic Analysis
March 18, 2017
Problems and Solutions

1. If x and y are odd integers, then which of the numbers below must be even?

A) xy **B)** $3x$ **C)** $2x+y$ **D)** $x+3y$ **E)** y^2

Answer D.

2. Lucia and her friends are registered in a social network. Here are Lucia's friends and their friends. (A line means friendship between two people. For example, Monica is Lucia's friend but Alex is not Lucia's friend.) If someone shares a photo with some of his/her friends then those friends can also comment on it. If someone comments a photo then all his/her friends can see the comment and the photo, but cannot comment on it unless they originally could.
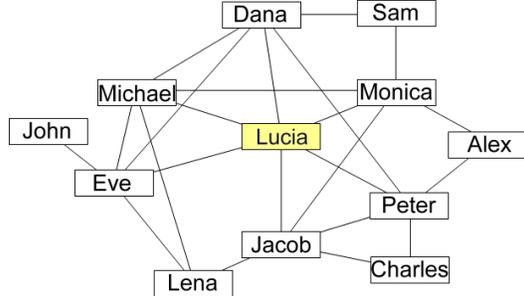
Lucia has uploaded a photo. With whom can she share it if she <u>does not want</u> Jacob to see it?

A) Dana, Michael, Eve **B)** Dana, Eve, Monica
C) Michael, Eve, Jacob **D)** Michael, Peter, Monica **E)** Another answer

Answer A.

3. Find the least real x such that $5 \leq |x| \leq 7$. **A)** 7 **B)** 6 **C)** 5 **D)** -5 **E)** -7

Answer E.

4. If $3a^3 + 6a^3 = 72$, then $9a^2 =$ **A)** 2 **B)** 4 **C)** 18 **D)** 36 **E)** 72

Answer D. We have $9a^3 = 72$, so $a^3 = 8$ and $a = 2$, hence $9a^2 = 36$.

5. Fred needs 2 hours to paint a fence, while Barney needs 3 hours for this. If they paint together, what part of the fence will be painted by Fred?

A) 3/5 **B)** 2/5 **C)** 2/3 **D)** 5/6 **E)** 3/4

Answer A. For $6x$ hours Fred can paint the fence $3x$ times, while Barney can paint it $2x$ times, so 3/5 out of these $5x$ times belong to Fred.

6. Stephanie found the binary watch shown below. She does not know how to figure out the exact time. Help her.
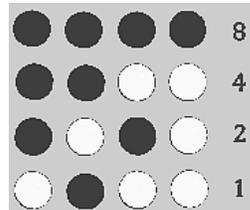
A) 01:34
B) 11:33
C) 12:23
D) 12:33
E) 12:57
Answer E.

7. The sum of nine consecutive <u>odd</u> integers is 657. The largest one among these integers is:

A) 65 **B)** 73 **C)** 77 **D)** 81 **E)** 85

Answer D. The average among these integers is $657:9 = 73$ and it equals the fifth integer. Then the largest one equals $73 + 4 \cdot 2 = 81$.

8. The perimeter of triangle ABC equals 4 times AC. If $AB=12$ and $BC=15$, then the angles α, β, γ of triangle ABC satisfy:

$\quad\quad$ **A)** $\alpha>\beta>\gamma$ $\quad$ **B)** $\alpha>\gamma>\beta$ $\quad$ **C)** $\beta>\alpha>\gamma$ $\quad$ **D)** $\beta>\gamma>\alpha$ $\quad$ **E)** $\alpha>\beta=\gamma$

Answer B. We have $4AC=12+15+AC$,

9. Two of the angles in a triangle measure 36º and 48º. Find the acute angle between the lines that bisect these two angles.

$\quad\quad$ **A)** 42º $\quad$ **B)** 54º $\quad$ **C)** 60º $\quad$ **D)** 84º $\quad$ **E)** 72º

Answer A. The angle in question is an exterior angle for a triangle, whose other two interior angles are 18º and 24º (the halves of the given angles)

10. A straight line passes through the points $A(1;9)$, $B(3;5)$ and $C(4;y)$. Find y.

$\quad\quad$ **A)** 0 $\quad$ **B)** 1 $\quad$ **C)** 2 $\quad$ **D)** 3 $\quad$ **E)** 4

Answer D. The slope of the line is $(5-9)/(3-1)=-2$, so $y=5-2=3$.

11. Mr. John has received a secret message. Unfortunately, a part of the message has been destroyed. In the figure to the right, destroyed cells are marked by ×. This case was foreseen and there are additional squares in the message. Each square in the rightmost column (column 6) or the lowest row (row 6) is coloured such that the number of black squares in a row, respectively in a column is even. What is the pattern of the destroyed piece?

A) $\quad$ **B)** $\quad$ **C)** $\quad$ **D)** $\quad$ **E)** None of these

Answer B.

12. Three identical jars were filled with water to 4/5 of their capacity. Then 1/4 of the water from each jar was poured out, and the remaining water in the three jars was put into two of the jars. Now the water is what part of the total capacity of these two jars?

$\quad\quad$ **A)** 11/20 $\quad$ **B)** 3/5 $\quad$ **C)** 22/30 $\quad$ **D)** 13/15 $\quad$ **E)** 9/10

Answer E. If the capacity of each jar is $5x$, then the water in it is $4x$ at the start and is $3x$ at the end. Then the total water is $9x$ and the total remaining capacity is $10x$.

13. Find the area of the triangle, whose vertices have coordinates $(0;0)$, $(8;6)$ and $(-6;8)$.

$\quad\quad$ **A)** 48 $\quad$ **B)** 50 $\quad$ **C)** 52 $\quad$ **D)** 54 $\quad$ **E)** 56

Answer B. The area of the rectangle with vertices $(-6;8)$, $(-6;0)$, $(8;0)$ and $(8;8)$ is $14 \cdot 8=112$. We need to cut from it two right triangles of total area $6 \cdot 8=48$ and a right triagle of area $14 \cdot 2:2=14$. The remaining area is $112-48-14=50$.

14. If $3x+2y=a$ and $3x-2y=b$, then $xy=?$

$\quad\quad$ **A)** $\dfrac{a^2-b^2}{2}$ $\quad$ **B)** $\dfrac{a^2-b^2}{4}$ $\quad$ **C)** $\dfrac{a^2-b^2}{6}$ $\quad$ **D)** $\dfrac{a^2-b^2}{12}$ $\quad$ **E)** $\dfrac{a^2-b^2}{24}$

Answer E. We have $x=(a+b)/6$, $y=(a-b)/4$.

15. In the school's computer lab the students have to set new passwords for their accounts. They are allowed to use lower-case letters, capital letters and the digits from 0 to 9. Each

password must have a typical order to be accepted. A-Z means any capital letter from the alphabet; 0-9 means any digit; a-z means any lower-case letter from the alphabet.
In a loop **any quantity** of letters or digits can be used several times.
The following loop allows **zero, one or more** capital letters:
An edge means **exactly one** letter or digit.
This edge demands one lower-case letter.
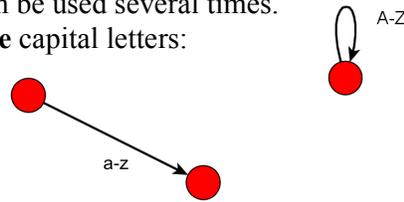
Which of the following passwords won't be accepted by the password machine shown to the right?
A) 123aNNa
B) Peter3ABCd
C) 2010Beaver4EVEr
D) bENNOZzz
E) Blagoevgr4d
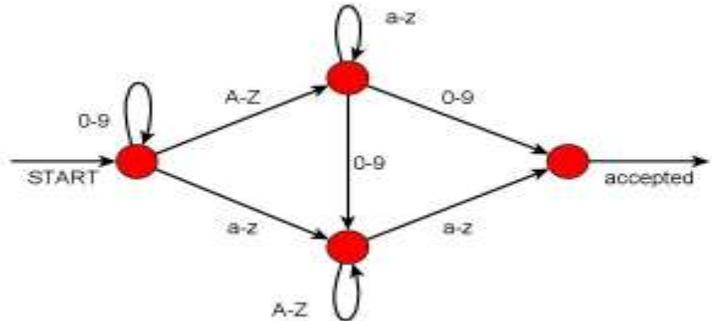
Answer D. "bENNOZzz" is the password that won't be accepted. It ends with a capital-case letter followed by two lower-case letters, which isn't allowed.

16. On the straight road from town A to town B there are gas stations and restaurants, numbered in direction from A to B. The distance between each two consecutive gas stations is 35 km. The distance between each two consecutive restaurants is 45 km. Gas station No.1 and Restaurant No.1 are in A. Find the number of the next gas station that is situated next to a restaurant.

A) 6   **B)** 7   **C)** 8   **D)** 9   **E)** 10

Answer E. We have LCM(35; 45)=315=9.35. After passing 9 intervals between gas stations one arrives at Gas station No.10.

17. How many hours are needed to mow a circular lawn of circumference 120π m, if it is mowed at a rate of 400π sq.m per hour?

A) 6   **B)** 8   **C)** 9   **D)** 12   **E)** 18

Answer C. The radius of the circle is 60, hence the area is $3600\pi=9\times400\pi$.

18. A certain price was increased by 25% and then the new price was increased by 40%. The same final price would be attained by a single increase of the initial price by:

A) 55%   **B)** 60%   **C)** 65%   **D)** 70%   **E)** 75%

Answer E. $1.25 \times 1.4 = 1.75$.

19. Find the perimeter of an isosceles triangle that has one side of length 4 and another of length 9.

A) 13   **B)** 17   **C)** 19   **D)** 22   **E)** It cannot be determined

Answer D. In order to have a triangle, the third side needs to be larger than the difference of the given two, so its length cannot be 4. Hence it is 9 and the perimeter is $9+9+4=22$.

20. The average of the measures of three of the angles of a pentagon is 98°. What is the average measure of the remaining two angles of this pentagon?

A) 113°   **B)** 118°   **C)** 123°   **D)** 128°   **E)** 133°

Answer C. The sum of these three angles is $3.98º=294º$, and the sum of the five angles is $3.180º=540º$. The remaining two angles have sum of $540º-294º=246º$, so their average measure is 123º.

21. In some community, people communicate using a whistle. Every message has its own code. During 10 seconds the person can use the whistle any odd number of consecutive seconds, and keep quiet any even number of consecutive seconds. The first and the last second of the 10-second period will always be used to whistle.
How many different messages can be made using these rules?

A) 10   **B)** 11   **C)** 15   **D)** 16   **E)** 256

Answer B. Explanation: 1000000001, 1000000111, 1000011111, 1001111111, 1110000001, 1110000111, 1110011111, 1111100001, 1111100111, 1111111001, 1001001001

22. The Game of Life is played on an infinite square grid. Some cells are populated (not empty). Two cells are called neighbors if they share a common side or a common vertex (so each cell has 8 neighbors). The next generation is created from the actual generation according to these rules:

➢ A populated cell survives if and only if it has two or three neighbors, otherwise it dies (on the figure, the grey cells with × die);

➢ An empty cell becomes populated if and only if it has 3 neighbors (as the grey empty cells do).
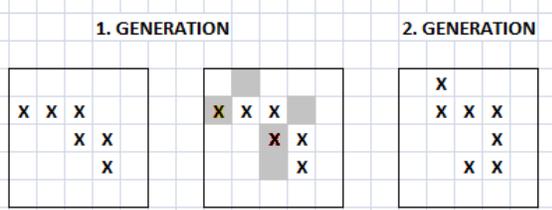
Which generation is correctly created from the 1-st generation shaped T ?
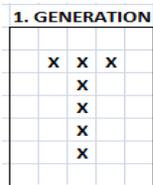
A)   B)   C)   D)
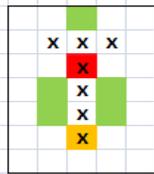
Answer B)

23. The average weight of four students is 53 kg. None of them weighs less than 46 kg. Each two differ by at least 3 kg. Find the largest possible weight of the heaviest student.
Answer 65. The least possible sum of the weights of the other three is $46+49+52=147$, while the sum of the four is $53.4=212$, so the needed weight is $212-147=65$ kg.

24. The points C, D lie on a circle with diameter AB. Find DB, if $AC=14$, $AD=16$ and $CB=8$.
Answer 2. By Pythagoras, $AC^2+CB^2=AB^2=AD^2+DB^2$, hence $DB^2=14^2+8^2-16^2=2^2$.

25. A right triangle has area 12 sq.cm and hypotenuse 11 cm. Find its perimeter in cm.
Answer 24. If the legs are a and b, then $(a+b)^2=a^2+b^2+2ab=11^2+2.2.12=121+48=169$, so $a+b=13$ and the perimeter is $13+11=24$ cm.

26. Find the largest three-digit number that has remainder 6 when divided by 7 and remainder 10 when divided by 11.

Answer 923. If x is the number in question, then $x+1$ is divisible by 7 and by 11, hence by 77. The largest three-digit multiple of 77 is $77 \cdot 12 = 924$, which yields $x = 923$, while the next multiple (1001) yields a four-digit number.

27. A car traveled with an average speed of 96 km/h from Sofia to Blagoevgrad. If on the way back its time was decreased by 25%, then its average speed has been increased by how many km/h?

Answer 32. The time is multiplied by 3/4, so the speed is multiplied by 4/3. Thus it is increased by 1/3, and 96/3 = 32.

28. A number sequence consists of the numbers 32, 16, 8, 4, 2, 1 and after that these 6 numbers are repeated forever. Find the sum of the 135th, 136th and 137th number in this sequence.

Answer 14. These numbers coincide with 3rd, 4th and 5th among these numbers and 8+4+2 = 14.

29. Ann is six years younger than Tom. Tom is now three times older than Ann. How many years from now will the age of Ann be equal to 75% of the age of Tom?

Answer 15. If Ann now is x, Tom is $3x = x+6$, hence $x = 3$, $3x = 9$. Their difference is 6 years and it will be equal to 25% of his age when he is $4 \times 6 = 24$. This will happen $24 - 9 = 15$ years from now.

30. Mike has won only 4 of his first 22 games in table tennis. What is the least number of additional games that he needs to play, knowing that he will lose at least 1/4 of them, in order to have won more than half of his games?

Answer 32. If he wins $3x$ games, he will lose at least x games. Then $4 + \underline{3x} > 18 + x$, so $\underline{2x} > 14$ and $\underline{x} > 7$. If $x = 8$, he will have played $4x = 32$ additional games.